Lab Assignment #2
Phase-sensitive Rectifier (Lock-in Amplifier)

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Objective

- Investigate the properties of an analog multiplier using time-independent and various time-dependent signals.
- Build the circuit of phase-sensitive rectifier and investigate its function.
- Measure the output signal of the phase-sensitive rectifier in dependence on the phase shift between the measured signal and the reference signal for both the auto-correlation and cross-correlation.
- Investigate the influence of noise superposed on the measured signal.
- Determine a unknown impedance using a phase-sensitive current voltage measurement.

Equipment

- Agilent 54600A Oscilloscope
- Agilent 33120A Function Generator
- Agilent 4284A RLC meter
- Agilent VEE software

A. Theoretical Introduction

A phase-sensitive rectifier realises a correlation method: The input signal is compared with a reference signal and the amount of similarity between these signals is determined. There are two kinds of correlation functions:

- The cross-correlation function

$$K_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} y(t)x(t+\tau)dt$$

  describes the similarity between two different time-continuous functions $y(t)$ and $x(t)$. If there is no similarity between the both functions, then the correlation function K_{xy} will be zero for all τ.

- The auto-correlation function

$$K_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} x(t)x(t+\tau)dt$$

  describes the independence of the function $x(t)$ with respect to a time shift τ and, therefore, determines the signal's similarity with itself. The following properties of the auto-correlation are characteristically:

  - At $\tau = 0$ the function has its maximum value, which is equal to the square of the RMS value: $K_{xx}(\tau = 0) = \overline{x^2}$.

- The auto-correlation function is an even function with respect to the time-shift τ : $K_{xx}(\tau) = K_{xx}(-\tau)$.

- At $\tau \rightarrow \infty$ the function is equal to the square of the mean value: $K_{xx}(\tau \rightarrow \infty) = \overline{x}^2$.

In a simple correlation measurement a mean builder determines the so-called product mean value of the measured and the reference signal, which is equal to the start value of the correlation function: $\overline{x(t) \cdot y(t)} = K_{xy}(0)$.
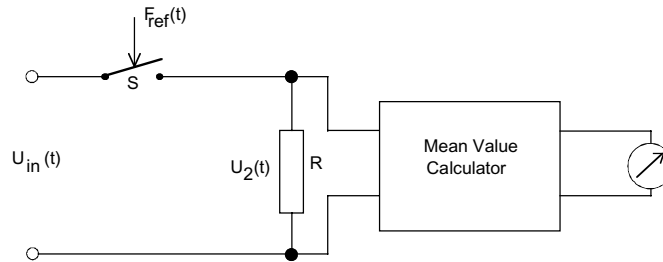

Fig.1:   Basic Principle of a Phase-Sensitive Rectifier

In Fig.1 the basic principle of a phase-sensitive rectifier is shown. If we use an input voltage $U_{in}(t) = \hat{U}_{in} \sin(\omega_{in}t + \varphi)$, and the switch S is opened and closed periodically with the frequency, ω , across the resistor R, the voltage $U_2(t) = U_{in}(t) \cdot F_{ref}(t)$ will be generated, where

$$F_{ref}(t) = \begin{cases} 1 & \text{for } 0 \leq t < T/2 \\ 0 & \text{for } T/2 \leq t < T \end{cases}$$

and $T = 2\pi/\omega$.

Developing $F_{ref}(t)$ into a Fourier's series and assuming $\omega_{in} = m\omega_{ref}$, we obtain

$$U_2(t) = \hat{U}_{in} \sin(m\omega t + \varphi)\left[\frac{1}{2} + \sum_{n=0}^{\infty} \frac{2}{(2n+1) \cdot \pi} \cdot \sin[(2n+1)\omega t]\right].$$

Because sine term inside the brackets average to zero, the mean value of this voltage is

$$\overline{U}_2 = \begin{cases} \dfrac{\hat{U}_{in}}{m\pi} \cdot \cos\varphi & \text{with } m = 2n+1 \\ 0 & \text{with } m \neq 2n+1 \end{cases}$$

If we replace the switch by a multiplier, we can use an harmonic reference signal. With

$$U_{in}(t) = \hat{U}_{in} \sin(\omega_{in}t + \varphi_{in}),$$
$$U_{ref}(t) = \hat{U}_{ref} \sin(\omega t + \varphi_{ref})$$

we obtain

$$U_2(t) = \frac{\hat{U}_{in} \cdot \hat{U}_{ref}}{2} \left[\cos(\omega_{in} - \omega)t \cdot \cos(\varphi_{in} - \varphi_{ref}) - \cos(\omega_{in} + \omega_{ref})t \cdot \cos(\varphi_{in} + \varphi_{ref}) \right]$$

and, furthermore, with $\omega_{in} = \omega$, $\varphi_{in} - \varphi_{ref} = \varphi$, $\varphi_{in} + \varphi_{ref} = \psi$

$$U_2(t) = \frac{\hat{U}_{in} \cdot \hat{U}_{ref}}{2} \left(\cos\varphi - \cos 2\omega t \cdot \cos\psi \right) .$$

Once more the output voltage consists of a DC voltage, which depends on the phase-shift between the measured and the reference signal and an AC voltage with the frequency 2ω. Also in this expression-only the first term contributes to the mean value; therefore we obtain

$$\overline{U}_2 = \frac{\hat{U}_1 \cdot \hat{U}_{ref}}{2} \cos\varphi .$$

Depending on the shape of the reference signal, we can distinguish between two different ways of using of the phase-sensitive rectifier:

1) Sine-wave-responding (fundamental responding): A harmonic reference signal is used; an output signal will be generated only at $\omega_{in} = \omega$, the measured value is proportional to the RMS.

2) Square-wave-responding (flat-responding): A square-wave reference signal is used; an output signal will be generated at $(2n+1)\omega_{in} = \omega$ (with $n = 0, 1, 2...$). Using a harmonic input signal the output signal is proportional to the RMS.

To understand the difference between these two methods consider the following example.

We use a square-wave input signal, which can be described by the following equation:

$$U_{in}(t) = \begin{cases} \hat{U}_{in} & \text{for } t_0 \le t < t_0 + T/2 \\ -\hat{U}_{in} & \text{for } t_0 + T/2 \le t < t_0 + T \end{cases}$$

1. Sine-wave responding:

Using a reference voltage $U_{ref}(t) = \hat{U}_{ref} \sin \omega t$, the mean value of the output signal is

$$\overline{U}_2 = \frac{\hat{U}_{in}}{T} \left[\int_{t_0}^{t_0 + \frac{T}{2}} U_{ref}(t)dt - \int_{t_0 + \frac{T}{2}}^{t_0 + T} U_{ref}(t)dt \right] = \frac{2\hat{U}_{in}\hat{U}_{ref}}{T} \int_{t_0}^{t_0 + \frac{T}{2}} \sin\omega t\, dt = -\frac{2\hat{U}_{in}\hat{U}_{ref}}{2\pi} \cos\frac{2\pi}{T} t \Big|_{t_0}^{t_0 + \frac{T}{2}}$$

$$= -\frac{\hat{U}_{in}\hat{U}_{ref}}{\pi} \left[\cos\frac{2\pi}{T}\left(t_0 + \frac{T}{2}\right) - \cos\frac{2\pi}{T} t_0 \right]$$

$$= -\frac{\hat{U}_{in}\hat{U}_{ref}}{\pi} \left[\cos(\varphi + \pi) - \cos\varphi \right] \quad \text{with} \quad \frac{2\pi t_0}{T} = \varphi$$

$$= \frac{2\hat{U}_{in}\hat{U}_{ref}}{\pi}\cos\varphi$$

and, with $\varphi = 0$, we finally obtain $\qquad \overline{U}_2 = \frac{2\hat{U}_m\hat{U}_{ref}}{\pi}$.

2. Square-wave responding:

Using a square-wave reference voltage $\quad U_{ref}(t) = \begin{cases} \hat{U}_{ref} & \text{for } \; t_0 \leq t < t_0 + T/2 \\ -\hat{U}_{ref} & \text{for } \; t_0 + T/2 \leq t < t_0 + T \end{cases}$

the mean value of the output voltage is

$$\overline{U}_2 = \frac{\hat{U}_{ref}}{T}\left[\int_0^{T/2} U_{in}(t)dt - \int_{T/2}^{T} U_{in}(t)dt\right]$$

$$= \frac{\hat{U}_{in}\hat{U}_{ref}}{\pi}(\pi - 2\varphi) \qquad \text{with} \qquad \varphi = 2\pi\frac{t_0}{T}$$

and with $\varphi = 0$ we obtain $\qquad \overline{U}_2 = \hat{U}_{in}\hat{U}_{ref}$.

In a special case, with $\hat{U}_{in} = \hat{U}_{ref} = 1$, we obtain $\overline{U}_2 = 1$.

In the measurement technology the following applications of the phase-sensitive rectifier are important:

1. Detection of strong-noisy signals.

2. Small-band detection of low harmonic signals, superposed with other signals having a greater amplitude but different frequency.

3. Phase-sensitive detection of harmonic signals superposed with phase-shifted signals with the same frequency.

B. Simulation Experiments

For better understanding of theory and usage of the phase-sensitive rectifier we use the simulation programs **PSR1.VXE** and **PSR2.VXE**. To run these programs you need the run-time version of Agilent VEE (version 5 or above). With these programs you can investigate the properties of a phase-sensitive rectifier used in the different modes and for the different measurement applications.

As pointed out already an important application of the phase-sensitive rectifier is the determination of the distance of an object by measuring of the travelling time of a reflected acoustic or electromagnetic signal . The principle of this method is demonstrated in the simulation experiment **PSR3.VXE**. The user can equalize the measured signal to a maximum of the product mean value by time-shifting of the reference signal. The distance, which is equivalent to this time-shift, will be given as number of wavelengths.

C. Laboratory Experiments

1. Exercises

1.1. Investigate the properties of an analog multiplier using time-independent and various time-dependent signals.

1.2.1. Build the circuit of phase-sensitive rectifier and investigate its function.

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1.2.2. Measure the output signal of the phase-sensitive rectifier in dependence on the phase shift between the measured signal and the reference signal for both the auto-correlation and cross-correlation.

1.2.3. Investigate the influence of noise superposed on the measured signal.

1.2.4. Determine a unknown impedance using a phase-sensitive current voltage measurement.

2. Theoretical Background

Beside the operational amplifier the multiplier is another important universal circuit for the analog measurement technology and signal processing. In normal operation the output signal is $y = k \cdot x_1 \cdot x_2$ with the input signals x_1, x_2 and the constant transfer factor k. Depending on the allowed range of the input signals the circuits are divided into

- single quadrant multiplier: $x_1, x_2 \geq 0$,
- two quadrant multiplier: $x_1 \geq 0$, for arbitrary x_2,
- and for quadrant multiplier: for arbitrary x_1, x_2.

The phase-sensitive rectifier (lock-in detector) is one of the possibilities to realise a correlation measurement method, which compares the measured signal with a reference signal and determines the similarity between them. The result is described with the correlation function.

Correlation methods are used for many different applications in the measurement technique, i.e.

- for measurement extremely noisy signals,
- for phase-dependent measurements of AC voltages, i.e. for impedance or admittance measurements ,
- for measurements of travelling times of acoustic or electromagnetic signals.

The basic principle of a circuit applicable for detection of noised signals as well as for phase-dependent measurements of an AC voltage is shown in Fig.1, page 2. The reference function $F_{ref}(t)$ opens and closes periodically with the frequency ω the switch S and therefore the signal

$$U_2(t) = U_{in}(t) \cdot F_{ref}(t)$$

across the resistor R will be produced . From this voltage the mean value

$$\overline{U}_2 = \begin{cases} \dfrac{\overset{\prime}{U}_1}{m\pi} \cdot \cos\varphi & \text{with } m = 2n+1 \\ 0 & \text{with } m \neq 2n+1 \end{cases}$$

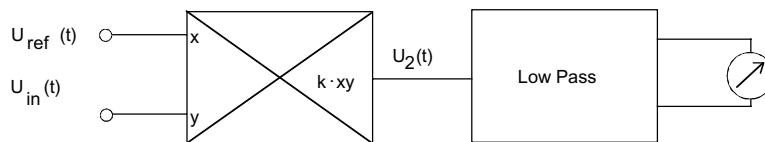will be determined by a low pass filter.

Fig.2: Principle of Calculation of the Product Mean Value

Using a multiplier instead of a controlled switch (Fig.2) the circuit generates directly the product mean value

$$\overline{U}_2 = \frac{\acute{U}_{in} \cdot \acute{U}_{ref}}{2} \cos \varphi \quad .$$

If the time constant of the low pass filter is high enough, a noise superposed on the measured signal will be strongly reduced.

A very useful method for resistance measurements is the current voltage method. This method can be used also for measuring complex AC resistors, if it is possible to measure the real and the imaginary part of the AC current independently. The basic circuit, using a current voltage converter, is shown in Fig.3, the output voltage of which is $\underline{U}^* = \underline{I}_x 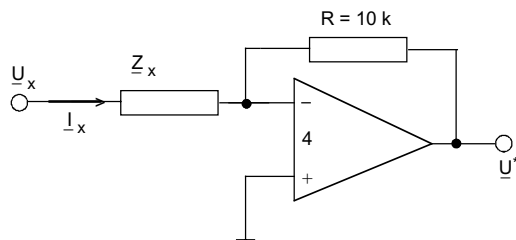\cdot R$. The current $\underline{I}_x$ is determined by the measured voltage $\underline{U}_x$ and the unknown admittance $\underline{Y}_x = 1/\underline{Z}_x$:

$$\underline{I}_x = \underline{Y}_x \cdot \underline{U}_x = (G_x + jB_x) \cdot \underline{U}_x .$$

With a phase-sensitive rectifier we can measure a voltage proportional to the real part of the current using an in-phase reference signal ($\varphi = 0$) as well as to the imaginary part using a 90 degree phase-shifted one ($\varphi = 90^o$). For the unknown admittance we obtain

$$G_x = \frac{\acute{U}^*_{0^o}}{U_x \cdot R} \quad , \qquad\qquad B_x = \frac{\acute{U}^*_{90^o}}{U_x \cdot R} \quad .$$

Fig.3: Current Voltage Converter

Keep in mind, that the phase-sensitive rectifier delivers the mean value

$$\overline{U} = \frac{\hat{U}^* \cdot \acute{U}_x}{2} \quad .$$

3. Instructions on the Laboratory Exercises

Caution: All circuits should be constructed using the *hps* Analog Development Board. Check conscientious the various circuits you build and determine there correct function by measurement and comparison of the signals at different circuit points using the oscilloscope Agilent 54600.

Before applying power to the circuit, please, ask the teacher to check the circuit for you.
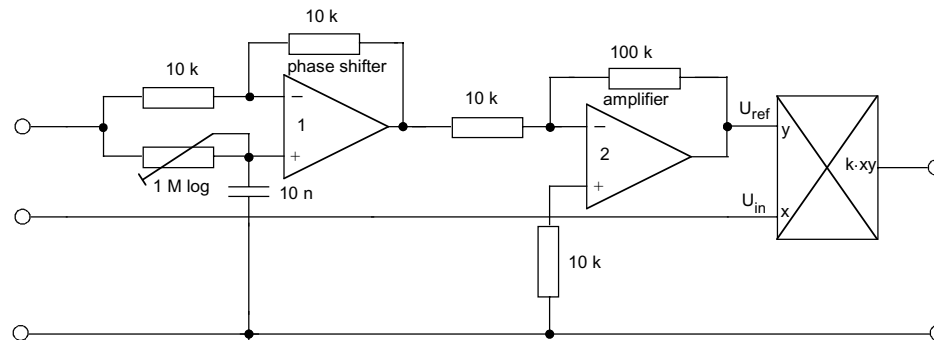
Exercise 1.1.:

Fig.4: Measurement Circuit for Investigation of the Multiplier with Phase Shifter and Amplifier

At first investigate the multiplier by measuring the output voltage using two independent controllable DC voltages. All voltages should be measured using DMM's. Determine the transfer factor k and the allowed input voltage range.

Than investigate the properties of the multiplier using time-dependent input voltages:

- $x_1 = U_1 = \text{const}, \; x_2 = U_2(t) = \acute{U}_2 \sin \omega t,$

- $x_1 = U_1(t) = \acute{U}_1 \sin(\omega t + \varphi), \; x_2 = U_2(t) = \acute{U}_2 \sin \omega t$ (in this case a phase shifter and an amplifier must be added as shown in Fig.4),

- $x_1 = U_1(t) = \acute{U}_1 \sin(\omega t + \varphi), \; x_2 = U_2(t)$: unipolare square-wave voltage.

You can generate controllable DC voltages with the potentiometers on the *hps* Development Board. For all time-dependent signals you need signal sources with very high frequency stability; use the function generator Agilent 33120A.

Exercise 1.2.:
For this exercise use the complete circuit of phase-sensitive rectifier shown in Fig.5. Determine the principle of function by measurement and comparison of signals at different circuit points.
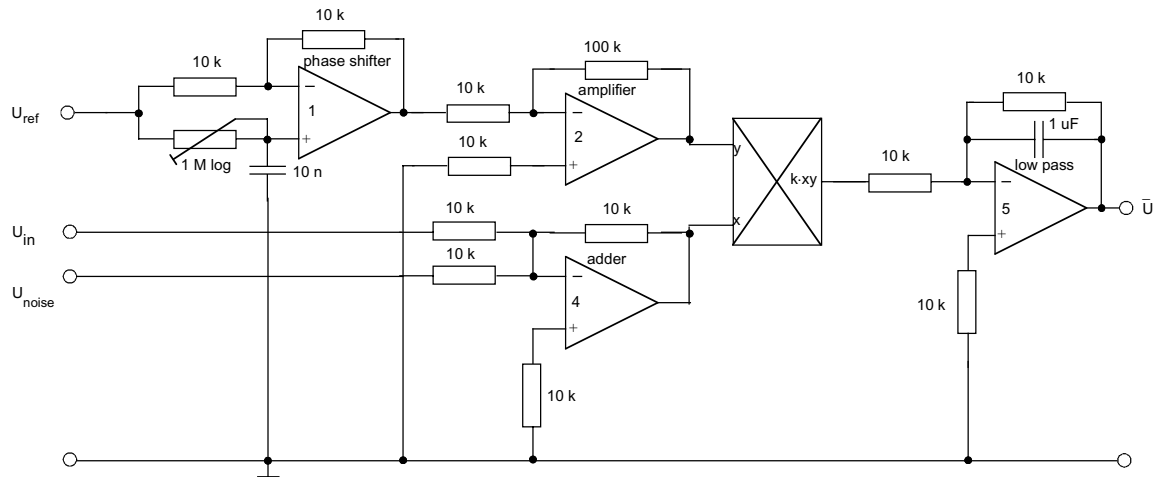
1.2.1.:

Fig.5: Phase-Sensitive Rectifier (Complete Circuit)

Construct the complete circuit on the *hps* Development Board and determine his function using the same voltage $U(t) = \acute{U} \sin \omega t$ for both signals U_{in} and U_{ref} (corresponding to auto-correlation; input U_{noise} short-circuited). Measure the signals in dependence of a phase shift φ at different circuit points using the oscilloscope. Measure the output signal $\overline{U}_2$ at a constant phase shift ($\varphi = 0$ or $\varphi = 180^o$) for various frequencies in a range $0,1\text{kHz} \le f \le 10\text{kHz}$.

1.2.2.:
Measure $\overline{U}_2$ in dependence on phase shift in a range $0 \le \varphi \le 180^o$ at a frequency $f = 1\text{kHz}$

1. In the case of auto-correlation (connect the inputs U_{ref} and U_{in} with one signal $U(t) = \acute{U} \sin \omega t$);

2. In the case of cross-correlation you need two signal sources with very stable frequency, and furthermore the frequency must be controllable in very fine steps. Use function

generators Agilent 33120 A; after a warm-up time of about 30 min the stability is high enough to start the measurements.

1.2.3.:
Use the circuit in auto-correlation operation. Superpose a noise voltage U_{noise} to the measured signal ($\acute{U}_{in} = 1V, \ f = 1kHz$); to generate the noise voltage use also a Agilent 33120 A. Measure the mean value with various amounts of noise ($0 \leq U_{noise} \leq 10V$).

1.2.4.:

Use the measurement circuit shown in fig.5 and use OpAmp4 on the *hps* Board as a current voltage converter (with $R = 10k\Omega$). As an unknown admittance use the parallel circuit of a resistor R_x ($1k\Omega \leq R_x \leq 10k\Omega$) a capacity C_x ($10nF \leq C_x \leq 50nF$).

Measure the output signal at $\varphi = 0$ (or $\varphi = 180^o$. Is this correct? - What is the difference?) and at $\varphi = 90^o$ and calculate the values R_x and C_x.

Measure the unknown admittance with the RLC meter Agilent 4284 A at the same frequency and compare the different results.